Micrometer and Nanometer Spatial Resolution with µPIV

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<td>PIV measurements of a microchannel flow</td>
<td>Meinhart CD, Wereley ST, Santiago JG</td>
<td>EXPERIMENTS IN FLUIDS 27 (5): 414-419 OCT 1999</td>
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<td>6</td>
<td>EFFECT OF RESOLUTION ON THE SPEED AND ACCURACY OF PARTICLE IMAGE VELOCIMETRY INTERROGATION</td>
<td>PRASAD AK, ADRIAN RJ, LANDRETH CC, OFFUTT PW</td>
<td>EXPERIMENTS IN FLUIDS 13 (2-3): 105-116 JUN 1992</td>
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<td>Low cost, high resolution DPIV for measurement of turbulent fluid flow</td>
<td>Fincham AM, Spedding GR</td>
<td>EXPERIMENTS IN FLUIDS 23 (6): 449-462 DEC 1997</td>
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<td>STEREOSCOPIC PARTICLE IMAGE VELOCIMETRY APPLIED TO LIQUID FLOWS</td>
<td>PRASAD AK, ADRIAN RJ</td>
<td>EXPERIMENTS IN FLUIDS 15 (1): 49-60 JUN 1993</td>
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<td>Iterative multigrid approach in PIV image processing with discrete window offset</td>
<td>SCARANO F, RIETHMULLER ML</td>
<td>EXPERIMENTS IN FLUIDS 26 (6): 512-523 1999</td>
<td>22</td>
<td>135</td>
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Figure 1 from Wereley and Meinhart, *Annual Reviews of Fluid Mechanics*, 2010 (Web of Science data as of November 2008).

*Purdue Microfluidics Laboratory*  25 Years of PIV, DLR Göttingen, 2009
Micro Particle Image Velocimetry (µPIV)

Santiago, Wereley, Meinhart, Beebe, Adrian, Exp. Fluids, 1998
US Patents 6,653,651 and 7,057,198--Licensed to TSI, Inc.

Micro-Fluidics Lab Purdue University

Nd:YAG Laser

Flood Illumination

Flow in

Flow out

Glass cover

Focal Plane

Epi-fluorescent Prism / Filter Cube

CCD Camera (1280x1024 pixels)

Micro-PIV image pair
Differences between $\mu$PIV and conventional PIV

- Brownian motion of nm-scale tracers
  \[ \varepsilon_B = \frac{\langle s^2 \rangle^{1/2}}{\Delta x} = \frac{1}{u} \sqrt{\frac{2D}{\Delta t}} \]
  where
  \[ D = \frac{\kappa T}{3\pi \mu d_p} \]

- Typically minimal optical access
  Volume illumination and wavelength filtering
  low particle concentrations

- Miniscule signal reflected from tracer particles
  Rayleigh scattering range ($d_p \leq \lambda$)
  A 100 nm particle scatters $10^6$ times more light than a
  10 nm particle
Where we started...

Santiago, Wereley, Meinhart, Beebee, and Adrian, “A particle image velocimetry system for microfluidics,” Exp. Fluids, 1998

Fig. 2a,b. Vector fields of a surface-tension driven Hele–Shaw flow around a 30 μm wide obstacle. Each field contains approximately 900 velocity vectors covering a 120 μm × 120 μm field of view. Each velocity vector was measured with a 6.9 μm × 6.9 μm × 1.5 μm measurement volume. a Instantaneous vector field measurement. b Eight-image ensemble-averaged PIV velocity vector field.
Correlation Analysis for μPIV (steady flow)

\[ R_{AB}(s) = \int A(X)B(X+s)dX \]

Three techniques involve the same operations
- 1. Acquire image fields
  - \textit{ensemble average}
- 2. Correlate image fields
  - \textit{ensemble average}
- 3. Determining velocity vector from peak in correlation
  - \textit{ensemble average}

Operations (2) and (3) are nonlinear and don’t commute.
Correlation of Ensemble-Averaged Image Fields

Image Sequence | Image A \((t = t_0)\) | Image B \((t = t_0 + \Delta t)\) | Correlation \(R_{AB}\) | Peak Search
---|---|---|---|---
1 | \(A_1\) | \(B_1\) |  |  |
2 | \(A_2\) | \(B_2\) |  |  |
3 | \(A_3\) | \(B_3\) |  |  |
\(\cdots\) | \(\cdots\) | \(\cdots\) | \(\cdots\) | \(\cdots\)
\(N\) | \(A_N\) | \(B_N\) |  |  |
(Ensemble Ave.) | \(< A >\) | \(< B >\) | \(R_{<A> <B>}\) |  

Purdue Microfluidics Laboratory 25 Years of PIV, DLR Göttingen, 2009
## Ensemble-Averaged Velocity Fields

<table>
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<tr>
<th>Image Sequence</th>
<th>Image A $(t = t_0)$</th>
<th>Image B $(t = t_0 + \Delta t)$</th>
<th>Correlation $R_{AB}$</th>
<th>Peak Search</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$A_1$</td>
<td>$B_1$</td>
<td>$R_{A_1B_1}$</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$A_2$</td>
<td>$B_2$</td>
<td>$R_{A_2B_2}$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$A_3$</td>
<td>$B_3$</td>
<td>$R_{A_3B_3}$</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N</td>
<td>$A_N$</td>
<td>$B_N$</td>
<td>$R_{A_NB_N}$</td>
<td></td>
</tr>
</tbody>
</table>

(Ensemble Ave.)
Ensemble-Averaged Correlation Function

Image Sequence

1

2

3

N

Image A
(t = t₀)

A₁

A₂

A₃

A_N

Image B
(t = t₀ + Δt)

B₁

B₂

B₃

B_N

Correlation
R_{AB}

R_{A₁B₁}

R_{A₂B₂}

R_{A₃B₃}

R_{A_NB_N}

< R_{A,B} >

(Ensemble Ave.)

Peak Search
Ensemble-Averaged Particle-Image Correlation Functions
Performance of 3 correlation methods
Meinhart, et al., JFE, 2000

Fig. 4  Comparison of the performance of the three averaging techniques: average velocity ●, average image □, and average correlation ▼

• Valid measurement is one which differs by less than 10% from the long-time averaged and smoothed vector field
Assessing Accuracy of µPIV

Measure a Known Flow

Top View

Side View

V(y,z)

300 µm

Measurement Area

30 µm

V(y,z)
Microchannel Flow (x-z plane)
wall-normal spatial resolution < 1um

Streamwise Position (µm)

Spanwise Position (µm)

0 5 10 15 20 25 30

0 40 50 60 70

10 mm/s

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Streamwise Profile (x-z plane)


2% FS error
Tracer Particle Diffusion

- Based on Brownian motion of tracers broadening correlation peak
- Einstein (1905) developed formula for diffusion coefficient

\[ \langle s^2 \rangle = 2D\Delta t \quad \text{where} \quad D = \frac{kT}{3\pi\mu d_p} \quad \Rightarrow \quad \langle s^2 \rangle = \frac{2\Delta tk}{3\pi d_p} \cdot \frac{T}{\mu(T)} \]
Calculate the particle image size $d_e$ in the object plane:

For light sheet PIV:

$$d_e = \sqrt{M^2 d_p^2 + d_s^2}$$

where $M$ is magnification, $d_p$ is particle diameter, and $d_s$ is spot size of imaging system.

For volume illumination (micro-PIV):

$$d_e = \sqrt{M^2 d_p^2 + d_s^2 + d_z^2}$$

$$d_z = \frac{zMD_a}{x_0 + z}$$
Relating Temperature to Peak Area Change
Olsen and Adrian, 2000

\[ \Delta s_0 = \frac{\sqrt{2}}{\beta} \sqrt{d_e^2 + 8M^2 \beta^2 D \Delta t} \]

where \( \beta^2 = 3.67 \) is fit parameter for matching Gaussian to Airy function

\[ \Delta A = \frac{\pi}{4} \left( \Delta s_{0,c}^2 - \Delta s_{0,a}^2 \right) = 2\pi M^2 \beta^2 D \Delta t \]

where \( \Delta s_{0,a} \) autocorrelation peak diameter (\( \Delta t=0 \))

and \( \Delta s_{0,c} \) is the cross-correlation peak diameter

\[ \frac{T}{\mu(T)} = \frac{3d_p}{2M^2 k \Delta t} = C_0 \frac{\Delta A}{\Delta t} \]
Temperature Measurement Results

\[
\frac{T}{\mu(T)} = \Delta A \frac{3d_p}{2M^2k\Delta t} = C_0 \frac{\Delta A}{\Delta t}
\]

- Measure correlation peak areas at e^{-1} level
- Use calibration to get constant of proportionality
- Original result within ±3°C over significant temp range (20-50 C)
- Recent results approx ±1.5°C over larger range (20-80 C)

Hohreiter, Chung, Olsen, Wereley, MST 2002
Chamarthy, Garimella, Wereley, Exp. Fluids 2009

Purdue Microfluidics Laboratory

25 Years of PIV, DLR Göttingen, 2009
Assess hydrodynamic size of particle
Kumar, Gorti, Shang, Lee, Yip, and Wereley, *J. Fluids Eng.*, 2008

- Use as biodetector for any number of substances
- Linear for small number of analytes per particle
- Sensitivity of about 1 virus per particle

\[ \zeta = 0.2705n + 5.8632 \]
\[ R^2 = 0.983 \]

700 nm particle with 10 M13 viruses attached
Single Pixel Evaluation (SPE)  
Westerweel, Geelhoed, Lindken, 2004

- With modern cameras and computers we can increase the sample number almost without bound...
- We can decrease the correlation region size to its smallest possible value: one pixel

\[
\Phi_{spe}(i, j; m, n) = \sum_{k=1}^{k_{tot}} f_k(i, j) \cdot g_k(i + m, j + n)
\]
Some Results (Westerweel, et al.)

- Infinitely thin shear layer (simulated)
- Flow in a nearly rectangular channel (experimental)
- Spatial resolution reported smaller than $d_p$

Fig. 7. The result for the profile of the displacement for an infinitely thin shear layer (32x32 px spatial correlation; single-pixel ensemble correlation)

Fig. 11. The mean velocity profile in the micro-channel obtained by means of 128x8-pixel ensemble correlation and single-pixel ensemble correlation
Experimental Parameters

Optics
- $M = 20x$, $NA = 0.4$, $\lambda \sim 0.5 \mu m$

Particle size
- $d_p = 0.5 \mu m$

Pixel size
- $d_{pix} \sim 300 \text{ nm}$

Diffraction spot size
- $d_{diff} \sim 1.3 \mu m$

Beating the Diffraction Limit

Wereley and Meinhart, 2005

Working well below the diffraction limit!

Ultimate limit, $M=100x$, pixel size $\sim 60 \text{ nm}$

<table>
<thead>
<tr>
<th>Particle Size</th>
<th>$M = 60$</th>
<th>$M = 40$</th>
<th>$M = 40$</th>
<th>$M = 20$</th>
<th>$M = 10$</th>
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</thead>
<tbody>
<tr>
<td>0.30 $\mu m$</td>
<td>0.42</td>
<td>0.69</td>
<td>0.98</td>
<td>1.28</td>
<td>2.93</td>
</tr>
<tr>
<td>0.50 $\mu m$</td>
<td>0.58</td>
<td>0.79</td>
<td>1.06</td>
<td>1.34</td>
<td>2.95</td>
</tr>
<tr>
<td>0.70 $\mu m$</td>
<td>0.76</td>
<td>0.93</td>
<td>1.17</td>
<td>1.43</td>
<td>2.99</td>
</tr>
<tr>
<td>1.00 $\mu m$</td>
<td>1.04</td>
<td>1.18</td>
<td>1.37</td>
<td>1.59</td>
<td>3.08</td>
</tr>
<tr>
<td>3.00 $\mu m$</td>
<td>3.01</td>
<td>3.06</td>
<td>3.14</td>
<td>3.25</td>
<td>4.18</td>
</tr>
</tbody>
</table>
What have I left out?

• Nearly everything—short talk…
• In recent years we’ve seen developed:
  – 3D systems
    • 3 hole mask
    • Diffraction pattern
    • Stereo
    • Astigmatism-based
  – Time-resolved systems
  – Evanescent wave PIV
  – Confocal PIV
• Important work on theory of μPIV:
  – Depth of correlation
  – Particle visibility
• And I’m still leaving a ton out…